Astronomical Polarization and ALMA Polarimetry

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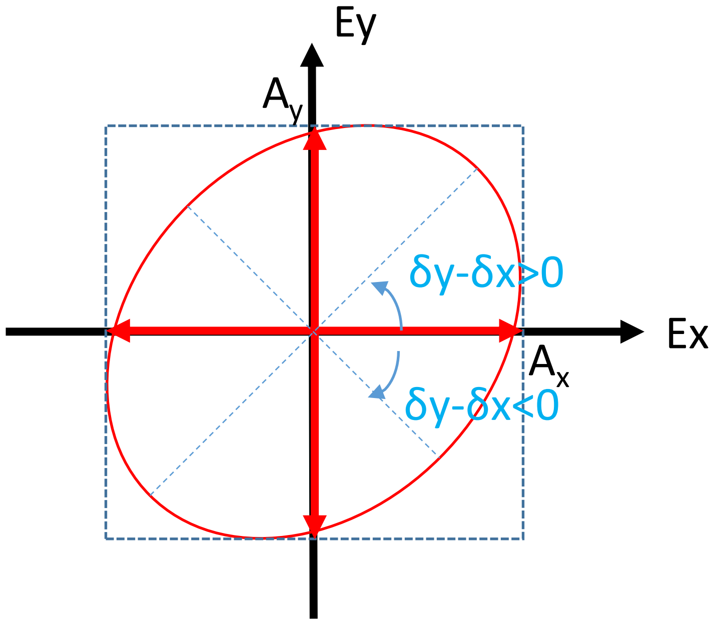
Electromagnetic (EM) Waves



- Propagation of electromagnetic field
- Characterized by wavelength, amplitude, and polarization.
- $\vec{E} \cdot \vec{k} = 0$, from Maxwell equations (E: Electric vector, k: wave vector)
- Mathematical expression

$$E_x = A_x \cos(2\pi\nu t + \delta_x)$$
$$E_y = A_y \cos(2\pi\nu t + \delta_y)$$

- δ_y-δ_x=0: linear polarization
- δ_y-δ_x=+/- π/2: circular polarization
- Otherwise: elliptical polarization

EM Waves in practice

- EM waves in real world are not monochromatic but a summation of multiple EM wave components.

$$E_x = \sum_i A_{x,i} \cos(2\pi\nu_i t + \delta_{x,i})$$

$$E_y = \sum_i A_{y,i} \cos(2\pi\nu_i t + \delta_{y,i})$$

- Thus, only partially or rarely polarized.
- Typically, less than few percent for most cases.
- Major polarization mechanism in radio astronomy:
  - Synchrotron radiation
  - Radiation from dust grain aligned to magnetic field or by radiation field
  - Scattering
  - Zeeman effect

Synchrotron radiation

- Interaction between relativistic electron and magnetic field
- $S_\nu \propto \nu^{-\alpha}$, thus brighter at lower frequency
- Linearly polarized
- EVPA $\perp$ B
- Useful to constrain projected magnetic field direction.

Synchrotron radiation

Orienti+ 2017

Contour: Stokes I
Color: Fractional Pol.

Two main spots are strongly polarized and parallel EVPA (perpendicular B-field), indicating strong shocks
Little polarization between two main spots and northern region where the emission is visible up to IR and optical, indicating turbulent acceleration plays a role for the diffuse emission

Prieto+ 2002

Dust polarization

Lazarian 2008

- Thermal radiation from dust grains aligned to B-field (Davis & Greenstein 1951)
- Linearly polarized
- EVPA ⊥ B
- Useful to constrain projected magnetic field direction.
- Note that polarization intensity is nothing to do with magnetic field strength.
- Similar alignment can be produced by radiative torque (RAT, e.g., Dolginov & Mitrofanov 1976).

Dust polarization

Rao+ 2000 (SMA observations)

Sadavoy+ 2018 (ALMA Observations)

Scattering

- Scattering by submm-mm size dust grains becomes dominant in ALMA frequencies.
- Linearly polarized
- Useful to constrain grain size (theoretical background: Kataoka+ 2015).
- NB: no clue for magnetic field

Kataoka+ 2015

Scattering

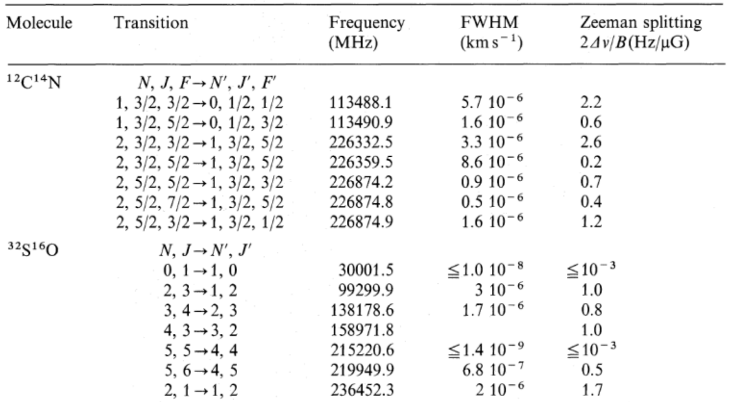
Kataoka+ 2016

- Polarization observation of protoplanetary disk HD 142527.
- Polarization vectors are radial in the main ring, while azimuthal in north-east.
- The flip of polarization vectors is predicted by the self-scattering of thermal dust emission due to the change of the direction of thermal radiation flux.

Zeeman effect

D. Crutcher's presentation

- Splitting of spectral line into different components under B-field
- One has RCP, another has LCP
- Frequency separation between RCP and LCP provides B-field strength
- In case radiation field is anisotropic in the source, the emission can be linearly polarized. (known as G-K effect)



Molecule	Transition	Frequency (MHz)	FWHM (km s^{-1})	Zeeman splitting $2\Delta\nu/B$(Hz/μG)
$^{12}C^{14}N$	$N, J, F \rightarrow N', J', F'$			
	1, 3/2, 3/2 → 0, 1/2, 1/2	113488.1	$5.7\ 10^{-6}$	2.2
	1, 3/2, 5/2 → 0, 1/2, 3/2	113490.9	$1.6\ 10^{-6}$	0.6
	2, 3/2, 3/2 → 1, 3/2, 5/2	226332.5	$3.3\ 10^{-6}$	2.6
	2, 3/2, 5/2 → 1, 3/2, 5/2	226359.5	$8.6\ 10^{-6}$	0.2
	2, 5/2, 5/2 → 1, 3/2, 3/2	226874.2	$0.9\ 10^{-6}$	0.7
	2, 5/2, 7/2 → 1, 3/2, 5/2	226874.8	$0.5\ 10^{-6}$	0.4
	2, 5/2, 3/2 → 1, 3/2, 1/2	226874.9	$1.6\ 10^{-6}$	1.2
$^{32}S^{16}O$	$N, J \rightarrow N', J'$			
	0, 1 → 1, 0	30001.5	$\leqq 1.0\ 10^{-8}$	$\leqq 10^{-3}$
	2, 3 → 1, 2	99299.9	$3\ 10^{-6}$	1.0
	3, 4 → 2, 3	138178.6	$1.7\ 10^{-6}$	0.8
	4, 3 → 3, 2	158971.8		1.0
	5, 5 → 4, 4	215220.6	$\leqq 1.4\ 10^{-9}$	$\leqq 10^{-3}$
	5, 6 → 4, 5	219949.9	$6.8\ 10^{-7}$	0.5
	2, 1 → 1, 2	236452.3	$2\ 10^{-6}$	1.7

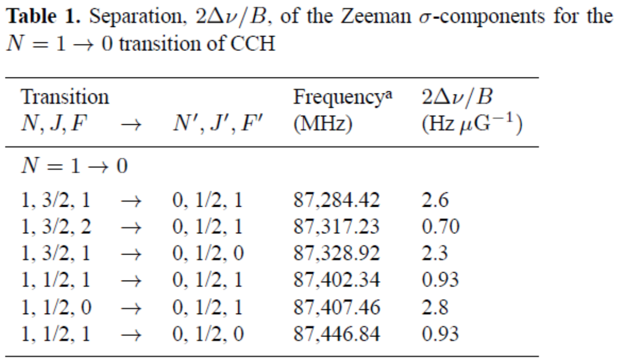


Table 1. Separation, $2\Delta\nu/B$, of the Zeeman σ-components for the $N = 1 \rightarrow 0$ transition of CCH

Transition N, J, F	→	N', J', F'	Frequency[a] (MHz)	$2\Delta\nu/B$ (Hz μG^{-1})
$N = 1 \rightarrow 0$				
1, 3/2, 1	→	0, 1/2, 1	87,284.42	2.6
1, 3/2, 2	→	0, 1/2, 1	87,317.23	0.70
1, 3/2, 1	→	0, 1/2, 0	87,328.92	2.3
1, 1/2, 1	→	0, 1/2, 1	87,402.34	0.93
1, 1/2, 0	→	0, 1/2, 1	87,407.46	2.8
1, 1/2, 1	→	0, 1/2, 0	87,446.84	0.93

Bel & Leroy 1989, 1998

Zeeman effect

- Observation of Orion using IRAM 30m
- Detection of CN Zeeman
- B_{los}=-0.36 +/- 0.08 mG on OMC1
- $V_{gas} \sim V_{alfven}$
- Gas motions are due to MHD wave

Crutcher+ 1999

Faraday rotation

- Propagation effect
- EVPA rotation in magnetized plasma
- Rotation angle ∝ λ^2
- RM ($\propto n_e B_{los}$) can be a clue for LOS B-field.

Graphs of polarization angle against wavelength squared for polarized extragalactic sources in the field (Haverkorn, Katgert, & de Bruyn 2003).

Faraday rotation

Marti-Vidal+ 2015

Hovatta+ 2019

5 x 10^5 rad/m^2 on quasar 3C 273

Very high RM (~10^8 rad/m^2) on quasar PKS 1830-211

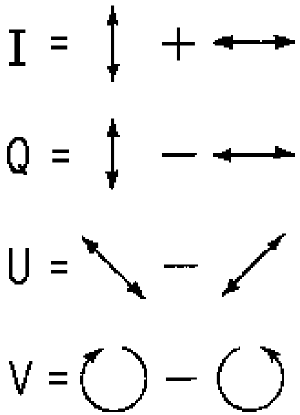
Stokes Parameter

$$I = \left\langle \left|E_x\right|^2 + \left|E_y\right|^2 \right\rangle$$

$$Q = \left\langle \left|E_x\right|^2 - \left|E_y\right|^2 \right\rangle$$

$$U = 2\left\langle \left|E_x\right|\left|E_y\right| \cos\ \phi \right\rangle$$

$$V = 2\left\langle \left|E_x\right|\left|E_y\right| \sin\ \phi \right\rangle$$



$$I = I_{unpol} + I_{pol} = I_{unpol} + \sqrt{Q^2 + U^2 + V^2}$$

$$LP = \sqrt{Q^2 + U^2}$$

$$\chi = 0.5 \tan^{-1}(U / Q)$$

$$CP = V$$

- Mathematical representation of polarization
- In the unit of power
- Q, U: linear polarization
- V: circular polarization

Polarimetry

X

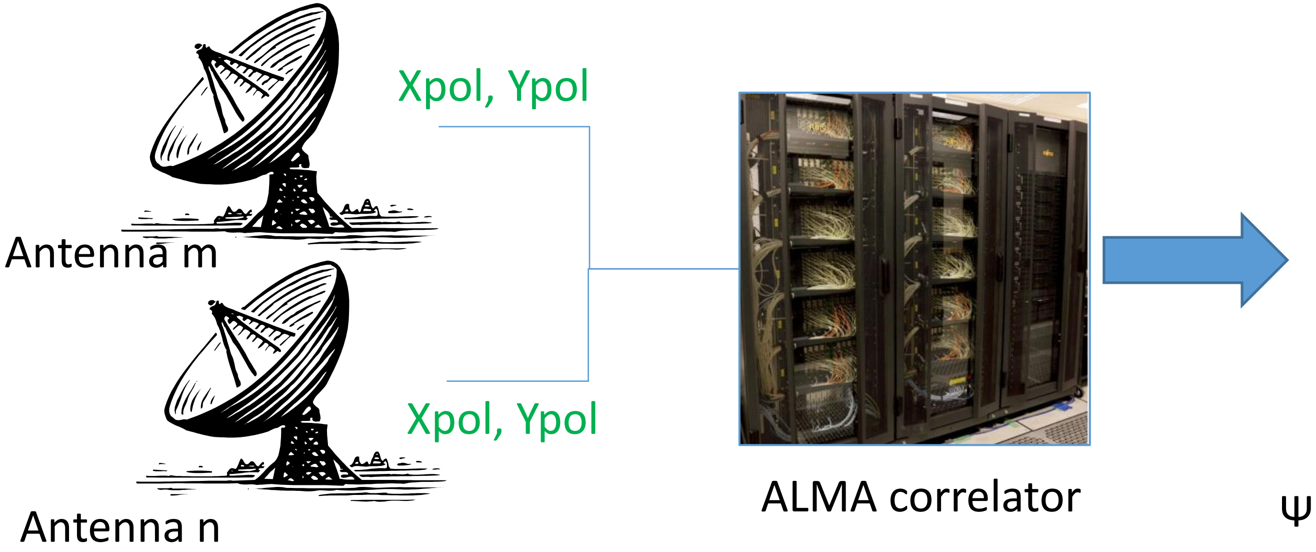
Y

- Polarization filter used in optical. Requiring rotation of the filter to measure the polarization direction.
- We can directly measure the amplitude and phase of two orthogonal linear polarization in radio, using such as cross-dipole antenna.

ALMA Frontend

Band 3/4/5/6/8: OMT
Band 7/9/10: Wire grid

ALMA Correlator


Xpol, Ypol

Antenna m

Xpol, Ypol

Antenna n

ALMA correlator

Cross correlation (XX, YY, XY, YX)

- 4 cross correlations per baseline

$$\hat{V}_{X_m}\hat{V}^*_{X_n} = I + Q\cos(2\psi_m) + U\sin(2\psi_m)$$

$$\hat{V}_{Y_m}\hat{V}^*_{Y_n} = I - Q\cos(2\psi_m) - U\sin(2\psi_m)$$

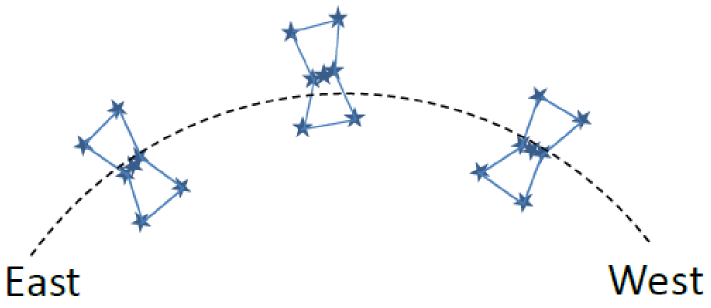
$$\hat{V}_{X_m}\hat{V}^*_{Y_n} = -Q\sin(2\psi_m) + U\cos(2\psi_m) + iV$$

$$\hat{V}_{Y_m}\hat{V}^*_{X_n} = -Q\sin(2\psi_m) + U\cos(2\psi_m) - iV$$

Ψ: parallactic angle

$$\psi(t) = \frac{\cos b \sin H(t)}{\sin b \cos \delta - \cos b \sin \delta \cos H(t)}$$

b = latitude; H(t) = Hour Angle; δ = declination



Instrumental polarization

$$\hat{V}_X = V_X + D_X V_Y$$

$$\hat{V}_Y = V_Y + D_Y V_X$$

The D terms: fraction of the input signal voltage in one polarization that leaks into the output of the other polarization. Typically few-several %.

$$\begin{cases} V_{X_i} V^*_{X_j} = G_{X_i} G^*_{X_j} \left[(I + Q_\psi) + U_\psi (D^*_{X_j} + D_{X_i}) + iV(D^*_{X_j} - D_{X_i}) \right] \\ V_{X_i} V^*_{Y_j} = G_{X_i} G^*_{Y_j} \left[U_\psi + I(D^*_{Y_j} + D_{X_i}) + Q_\psi (D^*_{Y_j} - D_{X_i}) + iV \right] \\ V_{Y_i} V^*_{X_j} = G_{Y_i} G^*_{X_j} \left[U_\psi + I(D_{Y_i} + D^*_{X_j}) + Q_\psi (D_{Y_i} - D^*_{X_j}) - iV \right] \\ V_{Y_i} V^*_{Y_j} = G_{Y_i} V^*_{Y_j} \left[(I - Q_\psi) + U_\psi (D^*_{Y_j} + D_{Y_i}) + iV(D_{Y_i} - D^*_{Y_j}) \right] \end{cases}$$

$$Q_\psi = Q \cos 2\psi + U \sin 2\psi,\; U_\psi = -Q \cos 2\psi + U \sin 2\psi,$$

Calibration of instrumental polarization

For simplicity, let’s assume V=0.

$$
\begin{cases}
V_{X_i}V^*_{X_j} = G_{X_i}G^*_{X_j}\left[(I+Q_\psi)+U_\psi(D^*_{X_j}+D_{X_i})\right] \\
V_{X_i}V^*_{Y_j} = G_{X_i}G^*_{Y_j}\left[U_\psi+I(D^*_{Y_j}+D_{X_i})+Q_\psi(D^*_{Y_j}-D_{X_i})\right] \\
V_{Y_i}V^*_{X_j} = G_{Y_i}G^*_{X_j}\left[U_\psi+I(D_{Y_i}+D^*_{X_j})+Q_\psi(D_{Y_i}-D^*_{X_j})\right] \\
V_{Y_i}V^*_{Y_j} = G_{Y_i}V^*_{Y_j}\left[(I-Q_\psi)+U_\psi(D^*_{Y_j}+D_{Y_i})\right]
\end{cases}
$$

- Using parallactic angle dependence, we can separate source polarization (Q, U) from D-terms.
- ALMA observes strongly polarized quasar as polarization calibrator over a wide range of parallactic angle. This is the reason why ≳3hr requires for polarization observations.

XY phase calibration

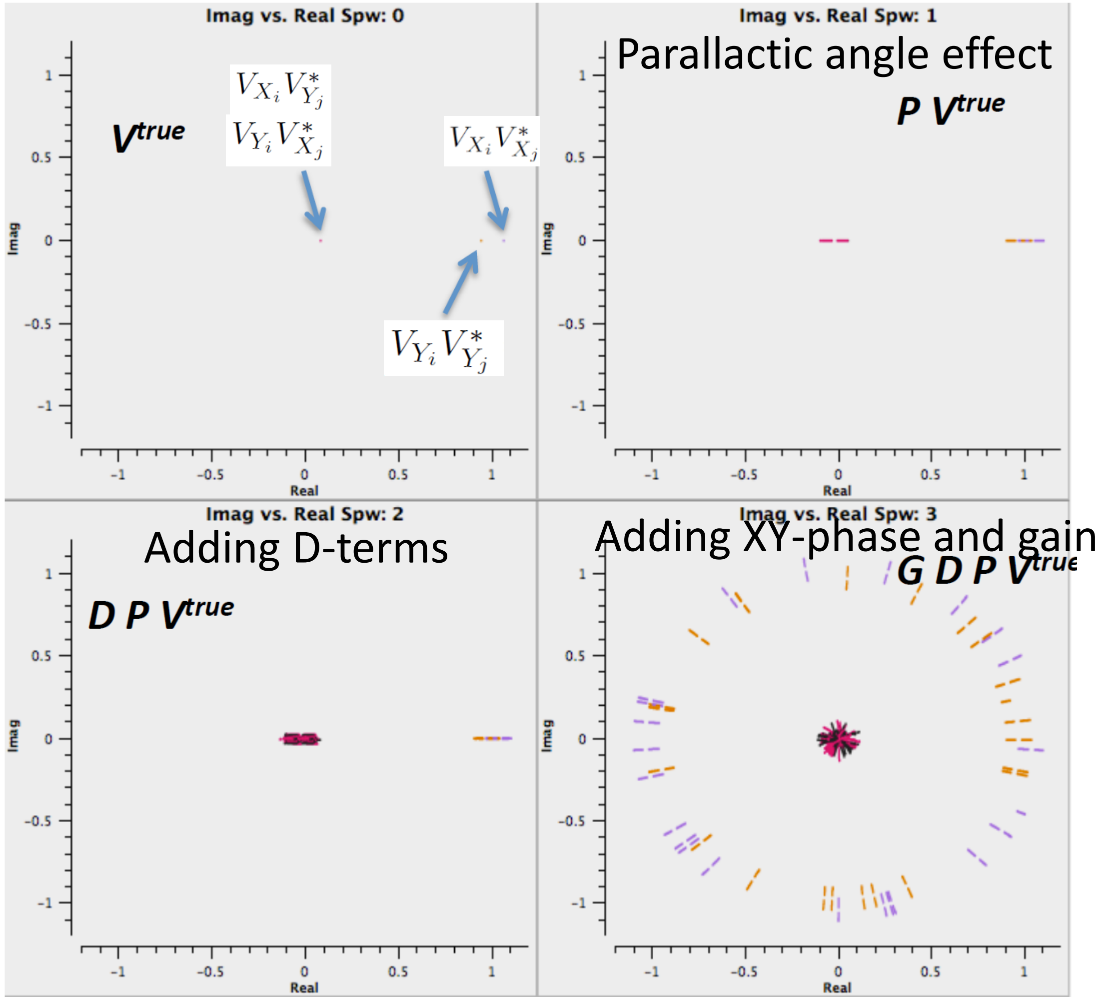
$$\begin{cases} V_{X_i}V_{Y_j}^* = e^{i\rho}\left[U_\psi + I(D_{Y_j}^* + D_{X_i}) + Q_\psi(D_{Y_j}^* - D_{X_i})\right] \\ V_{Y_i}V_{X_j}^* = e^{-i\rho}\left[U_\psi + I(D_{Y_i} + D_{X_j}^*) + Q_\psi(D_{Y_i} - D_{X_j}^*)\right] \end{cases}$$

$$\begin{aligned} \frac{\langle V_{X_i}V_{Y_j}^*\rangle + \langle (V_{Y_i}V_{X_j}^*)^*\rangle}{2} &= e^{i\rho}U_\psi + e^{i\rho}\left\langle I(D_{Y_j}^* + D_{X_i}) + Q_\psi(D_{Y_j}^* - D_{X_i})\right\rangle \\ &= e^{i\rho}U_\psi + \epsilon \end{aligned}$$

- Relative gain (Gx and Gy) can be calibrated by standard total intensity calibration, but the phase difference between X and Y for the reference antenna (ρ), so-called XY-phase, cannot be constrained.
- Obtain ρ by fitting <XY*>+<YX*> on complex plane.

Visualizing Polarization Effects (Simulation)

I=1, Q=0.06, U=0.08, V=0 (no noise)

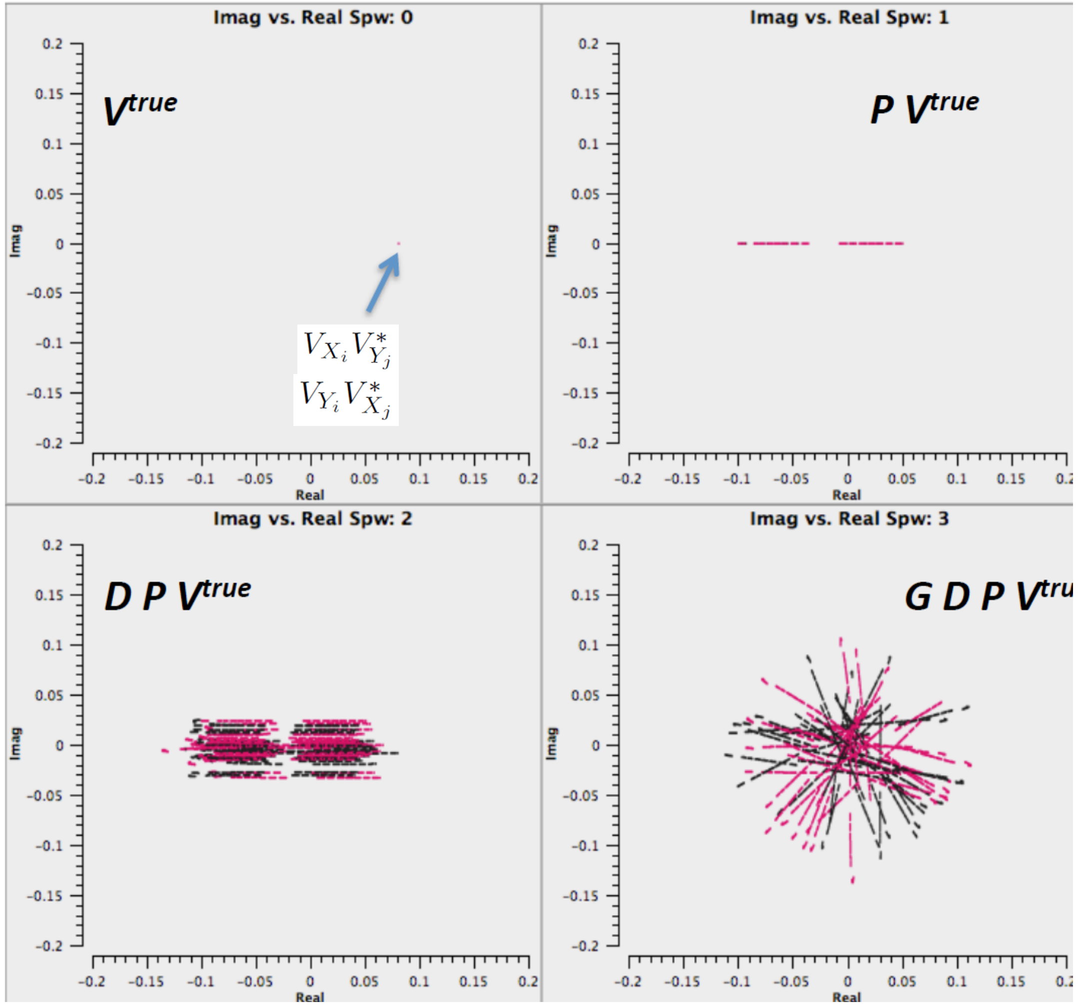


Parallactic angle effect

Adding D-terms

Adding XY-phase and gain

V_{XY}, V_{YX} only



Credit: G. Moellenbrock (NRAO)

Note for polarization imaging

- Deconvolution (CLEAN) needs to be done on I, Q, U, and V images separately.
- Q, U, and V can be negative, so do not forget to put CLEAN mask at negative emission components.
- Spatial structures are not always similar among I, Q, U, and V. Different shape of CLEAN masks should be applied on different Stokes parameters.

ALMA polarization capability

- Systematic error of linear polarization: ~0.1% at on-axis (<1/3 of PB)
  - Mainly because of calibration residuals of D-terms and XY-phase
- Systematic error of EVPA: ~1 deg at on-axis (<1/3 of PB)
- The errors increase to ~0.5% and 5 deg near the FWHM for linear polarization intensity and EVPA, respectively.
- Systematic error of circular polarization: ~1.8% within 1/10 of PB
  - Combination of multiple reasons

Data for hands-on tutorial

- Lin et al. 2024
- Band 4 continuum polarization
- Calibration have been applied.
- Let’s check the calibration quality together and work on imaging.